SHOCK WAVE IN A GAS – LIQUID MEDIUM

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The propagation of weak shock waves and the conditions for their existence in a gas–liquid medium are studied in [1]. The article [2] is devoted to an examination of powerful shock waves in liquids containing gas bubbles. The possibility of the existence in such a medium of a shock wave having an oscillatory pressure profile at the front is demonstrated in [3] based on the general results of nonlinear wave dynamics. It is shown in [4, 5] that a shock wave in a gas–liquid mixture actually has a profile having an oscillating pressure. The drawback of [3-5] is the necessity of postulating the existence of the shock waves. This is connected with the absence of a direct calculation of the dissipative effects in the fundamental equations. The present article is devoted to the theoretical and experimental study of the structure of a shock wave in a gas–liquid medium. It is shown, within the framework of a homogeneous biphasic model, that the structure of the shock wave can be studied on the basis of the Burgers–Korteweg–de Vries equation. The results of piezoelectric measurements of the pressure profile along the shock wave front agree qualitatively with the theoretical representations of the structure of the shock wave.

1. Structure of Shock Wave. It is shown in [6] that in the one-dimensional case the propagation of a low-frequency disturbance of finite amplitude in a gas–liquid mixture obeys the Burgers–Korteweg–de Vries equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \gamma\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^3 u}{\partial x^3} = 0 \tag{1.1}$$

where u is the perturbation velocity of the mixture.

In the case when the viscosity of the mixture is determined only by dissipative losses at the gas–liquid boundary, the coefficients of Eq. (1.1) take the form [7]

$$\gamma = \frac{2}{3}\frac{\nu}{\alpha_0(1-\alpha_0)}, \quad \beta = \frac{1}{6}\frac{R_0^2 c_0}{\alpha_0(1-\alpha_0)} \quad \left(\alpha_0 = \frac{4}{3}\pi R^3 N\rho\right)$$

Here ν is the kinematic viscosity coefficient, α_0 is the gas constant, ρ is the density of the mixture, N is the number of bubbles per unit mass, R_0 is the equilibrium radius of a bubble, and c_0 is the velocity of the low-frequency sound.

The stationary solutions of this equation can describe the structure of a weak shock wave [7, 8]. However, an examination of the structure of a shock wave in a gas–liquid mixture based on stationary solutions of this equation requires additional justification.

It is known [3, 7, 8] that in the process of the propagation of a disturbance dissipation balance the nonlinear effects and a stationary form of wave can be established – a shock wave front is formed. In a gas–liquid medium such losses can consist of dissipation in radial pulsations of the individual bubble because of slippage of bubbles in the wave front. It is shown in [5] that for shock waves whose width d considerably exceeds the bubble radius R the dissipation due to radial pulsations dominates over the dissipa-

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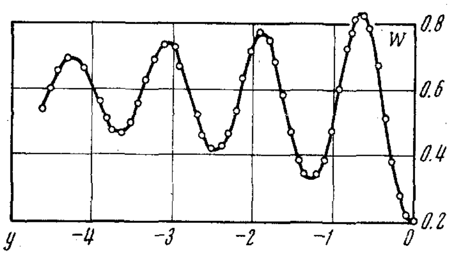

Fig. 1

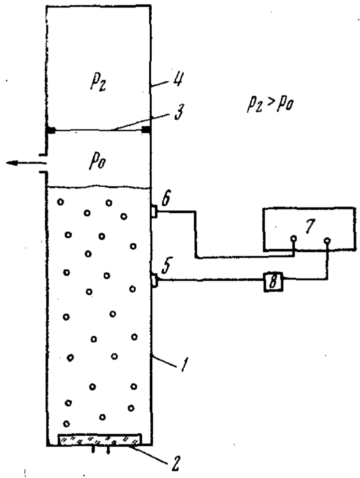

Fig. 2

tion due to slippage. This demonstrated the possibility of examining a shock wave on the basis of a model of a homogeneous medium.

Substituting $u = u\,(x - Vt)$ into (1.1) and integrating once [7] under the condition $u = u' = u'' = 0$ as $x \to \infty$, we obtain

$$\beta u'' - \gamma u' - u^2/2 - Vu = 0 \tag{1.2}$$

Restricted solutions of Eq. (2.1) describe the structure of a shock wave having a velocity $V = \Delta\, u/2$ in a reference system moving with velocity C_0, Δu being the jump in velocity. The result of a numerical calculation of the structure of a weak shock wave, which is preliminarily reduced to dimensionless form using the equations

$$W = u \,/\, c_0, \quad y = x - Vt \,/\, d, \; \beta_+ = \beta \,/\, d^2 c_0, \; \gamma_+ = \gamma \,/\, dc_0$$

conducted on the basis of Eq. (1.2), is presented in Fig. 1.

The shock wave propagates in the direction of positive values of x.

Figure 1 shows the results of the numerical solution of Eq. (2.1) at $\beta_+ = 10^{-2}$, $\gamma_+ = 5 \cdot 10^{-3}$, and $M = 1.3$. It is seen that the velocity in the shock wave has an oscillatory nature. At certain values of the dissipative and dispersive coefficients γ_+ and β_+ the shock wave will have a monotonic structure. The region of values of these coefficients corresponding to an oscillating shock wave profile can be determined from an asymptotic examination of the solutions of Eq. (1.2) as $y \to -\infty$ [7]. In this region the inequality $\gamma < (4\beta V)^{1/2}$ is satisfied, or in terms of a homogeneous model of a gas–liquid medium

$$\nu \,/\, R_0 c_0 < [{}^3/_2\, (M - 1)\; \alpha_0\; (1 - \alpha_0)]^{1/2} \quad (M = u_1 \,/\, c_0 = p_1 \,/\, p_0) \tag{1.3}$$

Here u_1 is the shock wave velocity in a stationary reference system, p_0 is the pressure in front of the shock wave, and p_1 is the pressure in the shock wave.

2. Experimental Study of Structure of Shock Wave. To study the pressure profile in a shock wave front we used the piezoelectric method of pressure measurement, which is characterized by low inertia, good sensitivity, and frequency-independent characteristics in the frequency range studied. Piezoelectric pickups with frequency-independent characteristics from 20 Hz to 50 kHz were used.

The schematic diagram of the experimental setup is shown in Fig. 2. The shock tube 1, 100 cm long with an inner diameter of 6 cm, was made of plastic. The gas–liquid mixture was produced by blowing nitrogen into the liquid through a porous glass filter 2 located in the lower part of the tube. We used PS-1 and PS-4 glass filters which produced bubbles 0.02 and 0.3 cm in diameter. Bubbles up to 0.6 cm in diameter wave obtained using a perforated disk. The radius of the fine bubbles was determined by photography.

The volumetric gas content α_0 of the mixture was varied from 0.01 to 0.15. The gas content was determined from the expression $\alpha_0 = h_1 - h_0/h_1$, where h_0 is the height of the column of liquid in the shock tube without gas bubbles, h_1 the height of the liquid column containing gas bubbles, and h_1 and h_0 were measured with a KM-3 cathetometer with an accuracy of ± 1 mm.

The disturbing pulse was produced by the rupture of diaphragm 3 which separated the high-pressure chamber 4 from the working part of the shock tube. One or several sheets of cellophane 0.04 mm thick were used as the diaphragm. The time of rupture of the diaphragm exceeded $2 \cdot 10^{-5}$ sec. It was estimated from the pressure rise time in the shock wave front propagating in the air. The pressure p_0 was always equal to atmospheric pressure.

Along the length of the shock tube, embedded flush with its wall, were arranged two piezoelectric pressure pickups of type LKh601. The signal from pickup 5 was fed to a cathode follower 8 and then to an oscillograph 7 of type PM-1, from whose screen the process was photographed. Pickup 6 served to trigger the oscillograph sweep.

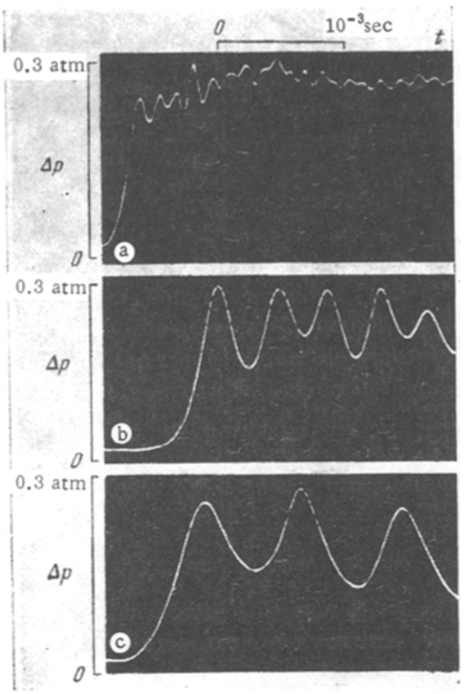

Fig. 3

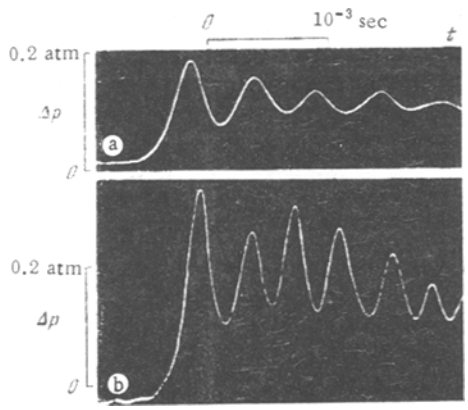

Fig. 4

A series of special experiments was conducted to measure the "noise" level of the rising bubbles, and the effect of the elastic waves generated in the wall of the shock tube on breaking the diaphragm was estimated. The signal distortions caused by these factors were on the order of the beam width on the oscillograph screen. The effect of reflected waves was eliminated by the choice of the sweep time, distance from the pickup to the bottom of the shock tube, and length of the high-pressure chamber. Reflected waves were registered by the pickup after a time interval greater than the duration of the sweep.

3. Results of Experiment. Oscillograms of the pressure in a shock wave front propagating in a gas–liquid mixture at an unchanged shock wave intensity as a function of the bubble radius are shown in Fig. 3a, b, c: a) $R_0 = 0.01$ cm, $\alpha_0 = 0.06$, b) $R_0 = 0.15$ cm, $\alpha_0 = 0.03$, c) $R_0 = 0.3$ cm, $\alpha_0 = 0.06$. The viscosity of the liquid is $\nu = 10^{-2}$ cm/sec and the Mach number is $M = 1.12$. The increase in frequency of the oscillations in the shock wave front and the decrease in their amplitude with a decrease in bubble radius are clearly seen in Fig. 3a, b, c.

The dependence of the amplitude and frequency of the oscillations in the shock wave front on its intensity is presented in Fig. 4a, b. Here $\alpha_0 = 0.06$, $R_0 = 0.15$ cm, $\nu = 10^{-2}$ cm/sec.

Shock waves with an oscillating pressure profile were not observed in the experiments of [1]. This is evidently connected with the fact that in the experiments the authors used a 50% solution of glycerin in water, and an analysis of the other parameters of their experiment on the basis of (1.3) indicates that under these conditions the shock wave has a monotonic profile.

The existence of shock waves with an oscillating pressure profile is possible with certain relationships of the parameters of the shock wave and the gas–liquid medium. The criterion for the existence of a shock wave with oscillating pressure in such media is established.

The frequency and amplitude of the pressure oscillations in the shock wave front depend on its intensity, the bubble diameter, and the viscosity of the liquid.

The experimental results obtained agree qualitatively with the theoretical representations of the shock wave which follow from the homogeneous model of a gas–liquid medium. It should be noted that analogous experimental results were obtained in [10].

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